A Novel Approach to Teaching and Assessing Students' Critical Thinking in University Mathematics

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This paper investigates the attitudes of the university mathematics lecturers towards the use of deliberately misleading mathematics questions in teaching and assessment as a pedagogical strategy with their students. The intention of using such questions is to enhance students' critical thinking. Critical thinking is understood here as "examining, questioning, evaluating, and challenging taken-for-granted assumptions about issues and practices" as defined by the New Zealand Ministry of Education. The study is based on a survey of university lecturers who have been introduced to the suggested pedagogical strategy. Their attitudes are compared with the attitudes of secondary school mathematics teachers.

Keywords: critical thinking, assessment, university mathematics

INTRODUCTION AND RATIONALE

The simple reality is that we face errors, mistakes, biases, fakes, contradictions on a regular basis in our lives. They are everywhere including the media, politics, and business. We are moving from the age of information to the age of misinformation. Some slips are genuine mistakes, but many are intentional aimed to mislead, manipulate, exploit, or deceive us. To avoid this, we should be critical and sceptical in a good sense of these words. As educators, we should teach our students to think critically by evaluating, analyzing, and challenging any situation or information, questioning the question, and not taking anything for granted, which is an essential part of mathematical thinking. In mathematics, we can be more proactive with developing and enhancing students' critical thinking skills by including specific activities dealing with *recognising* mistakes and errors. Mathematics is not immune from mistakes. Apart from the common practice of analysing and correcting students' mistakes, a powerful activity with students is finding and discussing mistakes in mathematics textbooks, journals and even exams.

An innovative pedagogical strategy was suggested by Klymchuk (2015) to prepare students and teachers to address mathematical mistakes and enhance their critical thinking, and this approach was further developed in Klymchuk & Sangwin (2021) and Klymchuk (2022). The idea was to use so-called provocative questions. A provocative question is a question that is *deliberately* designed to mislead a solver. It looks like a typical routine question, but it has a catch. It is often formulated as a possible task whereas the task is impossible. For example, the question might ask for a proof of something that is not provable; or show the existence of a solution of an equation that does not have a solution; or find a certain feature or characteristic (e.g. area, derivative) of a mathematical object (e.g. triangle, function) in a case where the object itself does not exist; or choose a correct answer in a multichoice format where *all* suggested answers are incorrect. Often a catch is based on a restricted domain or indirectly prompts the use of a rule, formula, or theorem that is inapplicable due to their conditions/constraints.

The intention of using provocative questions is to enhance students' critical thinking by encouraging them to pay attention to detail, question the question and eventually transfer this skill to any situation in life. Sometimes erroneous statements are being used in teaching and assessment. However, they are normally clearly labeled as incorrect statements, and the question asks students to correct them. For example, a question might ask to critique a misleading graph, disprove a false statement by providing a counterexample, identify a mistake in a sophism, or verify whether the given statement is correct or incorrect and justify the answer. These are good questions that test students' conceptual understanding and critical thinking. However, they contain prompts and hints that something is wrong, which alerts students to think critically and act differently compared to solving a standard procedural question. However, in real life there are often no prompts and hints about coming errors or fakes. Therefore, the provocative questions go one step further than the questions on incorrect statements, with prompts that ask for critique or disproof. They demonstrate the importance of always being alert and ready to analyse everything.

One of the findings from the study by Klymchuk (2015) and the replicated study by Brown (2018), both conducted with school mathematics teachers, was that 60% of the 127 participants reported that they would include provocative questions in their teaching. The remaining 40% of participants clearly indicated that they would not use provocative questions, mainly because they were not part of the assessment. Participants in another study (Klymchuk, 2022) with 82 school mathematics teachers were more optimistic about using provocative questions in their teaching. Ninety-six percent indicated that they intended to include provocative questions in their teaching, as they clearly saw benefits for their students in terms of enhancing their critical thinking. However, only 63% of the participants agreed to include provocative questions in the assessment. The remaining 37% of participants expressed two common concerns – additional stress for students and the need for preparedness for such questions.

This paper extends the above studies with school mathematics teachers to university lecturers. It investigates university mathematics lecturers' attitudes towards the use of provocative mathematics questions in their teaching and assessment.

THEORETICAL CONSIDERATIONS

As a general theoretical framework, a theory about education developed by Henry J. Perkinson is adopted in the paper. Perkinson is a renowned expert in educational history and theory, who published 13 books on these topics. He describes the essence of his theory:

The growth or evolution of students' knowledge is a matter of modifying the knowledge they already possess. Students—and all human beings—create their knowledge, just as all organisms create their progeny. We create theories about the world we inhabit and formulate skills to cope with that world. But because we are fallible, the knowledge and behaviours we create are always imperfect in some way—mistaken, wrong, erroneous, inadequate. Yet, at the same time, because our knowledge is imperfect, it can always improve, grow, and become better. Criticism facilitates this growth. Criticism can uncover some of the inadequacies in our knowledge, and when we eliminate them, our knowledge evolves and gets better…Education, thus, is a continual process of trial and-error elimination. Students are fallible creators who make trial conjectures and formulate trial skills and then eliminate the errors uncovered by criticism or critical selection. (Perkinson, 2002, pp. 365-366).

The notion of inquiry aligns well with Perkinson's concept of students' knowledge growth or evolution. Laursen and Rasmussen (2019) consider inquiry as a branch of active learning with the distinguishing characteristic of offering "students and instructors greater opportunity to develop a critical stance toward previous, perhaps unquestioned learning and teaching routines" (p. 132). By a critical stance they accept the following definition: "an attitude or disposition towards oneself, others and the object of inquiry that challenges and impels learners to reflect, understand and act in the milieu of potentiality" (Curzon-Hobson, 2003, p. 201).

This strategy is supported by The Critical Thinking Consortium (2013), which claims that "when students think critically in mathematics, they make reasoned decisions or judgments about what to do and think. In other words, students consider the criteria or grounds for a thoughtful decision and do not simply guess or apply a rule without assessing its relevance".

THE STUDY

This (pilot) study was conducted with eight experienced university mathematics lecturers from four universities: six lecturers from New Zealand, one from the UK and one from Australia. All lecturers had PhD either in mathematics or a related field. A combination of judgement and convenience sampling methods was used to select the participants. The participants gave their permission for the use of the survey results for dissemination among colleagues and research communities.

The research question was to analyse the reasons for a possible adoption or otherwise of the proposed use of provocative mathematics questions in teaching and assessment in university mathematics courses. All lecturers have been familiarized with the suggested pedagogical strategy via attending seminar presentations or reading articles on the topic. They were also given several examples of provocative questions from calculus. Three such provocative questions are below.

Question 1. *Find the derivative of the function* $y = \ln(\ln(\sin x))$.

Although it appears to be a routine question on differentiation techniques using the Chain Rule, the rule is not applicable because the function has an empty domain; therefore, the derivative does not exist.

Question 2. *Sketch a graph of a function that is differentiable on the interval* (a,b) *and discontinuous at least at one point on* (a,b).

Any sketch would be incorrect as the task is impossible: a function differentiable on interval (a,b) is continuous on it.

Question 3. *Find the following definite integral using the Newton-Leibniz formula:* $\int_{-1}^{1} \frac{dx}{x}$.

This question looks like a routine question on integration, but this is not a definite integral since the integrand function is not continuous on the interval [-1, 1]. Therefore, the Newton-Leibniz formula is not applicable.

The participants were given the following short questionnaire.

Questionnaire.

Question 1. Would you agree to use provocative questions in the teaching of your mathematics courses?
Yes No Why?

Question 2. Would you agree to use provocative questions in the assessment of your mathematics courses?
Yes No Why?

Participants' Responses to Question 1

Six out of eight lecturers (75%) reported that they would include provocative questions in their teaching. Five lecturers connected the use of such questions with improving students' critical thinking abilities. Four lecturers commented on the likely transferability of such abilities into the 'real' life outside of university including making better decisions and becoming more responsible citizens. There were three comments on expectations of the increase of students' engagement due to the "unusual", "thought provoking" and "entertaining" nature of the provocative questions. In follow-up informal interviews, all six lecturers agreed that a good introduction and explanation are needed before using provocative questions in teaching. This is mainly to avoid confusion among students, especially those who are non-native English speakers. One lecturer commented that the strategy might be less effective for weaker students, but it was still worth trying. Two out of eight lecturers (25%) reported that they were not willing to include provocative questions in their teaching. One lecturer commented on the lack of time due to the full content of his courses. The other lecturer expressed a concern that such questions might confuse some students and should be avoided.

Participants' Responses to Question 2

Three out of eight lecturers (38%) reported that they might try including provocative questions in the assessment. They clearly saw benefits for students from such questions and indicated that if the questions were taught in class they should be in the assessment. One lecturer however commented that he would warn his students by marking provocative questions with a star or red colour font. Five lecturers out of eight (62%) reported that they would not include provocative questions in the assessment. Three lecturers were concerned that such questions might lower the pass rate and generate negative comments from university management. Two lecturers were concerned about possible complaints from their students as such questions might be perceived as "unfair" or "abnormal" by some students.

CONCLUSIONS AND FURTHER STUDIES

The attitudes of the university mathematics lecturers from this study and school mathematics teachers from the three studies by Klymchuk (2014, 2022) and Brown (2018) towards the use of provocative questions in their courses were similar. Generally, very positive responses were given regarding the students' benefits from such questions in terms of enhancing their critical thinking; however, concerns were expressed about using them in assessments. However, the reasons for not willing to use such questions in the assessment were different. School teachers were more concerned about their students (unpreparedness, additional stress) whereas lecturers were more concerned about a possible negative impact on their own career. There were just a few comments from school teachers (209 teachers in total from the three studies above) regarding the transferability of critical thinking skills developed in the classroom to other domains of everyday life outside of school. They saw their job mainly as preparing their students for further studies – the next year at secondary school or at the tertiary level. Whereas half of the lecturers surveyed strongly linked critical thinking skills to students' future employment, career, and citizenship. They believed that critical thinking would help their students make better decisions and become more informed citizens.

The acronym VUCA (volatility, uncertainty, complexity, and ambiguity), introduced in 1987, is gaining popularity among politicians, business leaders, and scientists to describe the world we live in. The ambiguity refers to the potential for misreads and the mixed meanings of conditions. Regarding ambiguity, Bodenhausen and Peery (2009) note that sociologists focus on details such as whether sufficient information was present and whether the subject had the full amount of knowledge necessary to make a decision, and why they came to their specific answer. It directly relates to education in general and critical thinking in particular. To investigate the issue of transferring critical thinking skills from a mathematics classroom into other life situations, particularly the workplace, would require a series of longitudinal studies.

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